Angle domain illumination analysis along a target horizon
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Summary
We develop an efficient scheme of illumination analysis along a target horizon. With this scheme, we can calculate the Directional Illumination (DI) from the sources and the Acquisition Dip Response (ADR) along a target horizon in very short turnaround time. Therefore, it can be a useful tool to study the influence of the model (e.g. salt body) and the acquisition system (e.g. shot distribution and aperture size). The result can be a guide for acquisition design and model building. With the illumination map along the target horizon, it also is helpful for the interpretation in areas where the image amplitudes are not reliable. Here, we use the wave-equation based migration and local plane wave decomposition method to get the frequency domain illumination in the local angle domain. We pre-calculated and saved the angle domain Green’s function along the target horizon. These Green’s functions are reusable so that we can save a lot of computational and I/O cost. We use the 3D SEG/EAGE salt model and a real model example to demonstrate the validity of our method.

Introduction
Seismic illumination analysis is a useful tool that gives potential detecting power of a specific acquisition system for a given subsurface structure. For example, if there’s a huge salt body in the survey area, which is very common in Gulf of Mexico, we usually have illumination issues in the subsalt area. The illumination analysis can give us an estimation of the energy distribution in those areas, which can help us improve our acquisition design and get better understanding of the migration image. Both ray tracing (Bear et al., 2000) and wave-equation based illumination analysis methods (Rickett, 2003; Jin and Xu, 2010) have been developed in recent years. Ray-based methods are efficient, but will have large errors when the model is complex. Therefore we prefer wave-equation based methods.

Because the subsurface structures usually have certain dipping angles and the total illumination cannot give us any angle information, a lot of effort has been made towards angle domain illumination (Wu and Chen, 2006; Xie et al., 2006; Cao and Wu, 2009). Angle decomposition is very expensive, especially in the 3D case. To make the calculation more efficient, we made some implementations for high performance illumination analysis in the 3D case (Mao and Wu, 2007; Mao et al., 2010).

The source side illumination (DI) is relatively easy to calculate. However, the illumination of a source coupled with receivers is more important and is a more accurate estimation of the subsurface illumination for a given acquisition system. We call this kind of illumination Acquisition Dip Response (ADR). A similar approach using ray-tracing has been reported by Lecomte (2008), and is widely used through NORSAR software. However, if the target horizon is in a complex area such as subsalt, a wave-equation propagator is more accurate.

In order to calculate ADR, we need to calculate the Green’s function for every receiver, which is very expensive for a large model and large data set. In a real case, we usually focus on a horizon which could be a reservoir seal. As a result, we proposed to calculate the illumination along a target horizon, which is affordable and efficient.

In this paper, we develop an efficient scheme of illumination analysis along a target horizon. Here, we use the wave-equation based migration and local plane wave decomposition method to get the frequency domain illumination in the local angle domain. Since we pre-calculated and saved the angle domain Green’s function on the target horizon, we only need to form the illumination by the summation of the Green’s function combination for the given acquisition system. This scheme provides a useful tool for acquisition design and analyzing the image amplitudes. We calculated several numerical examples including the DI and ADR maps for the 3D SEG/EAGE salt model and a real model on some given target horizons.

Illumination analysis in the local angle domain
For a given acquisition geometry, we use a wave-equation based propagator to get the frequency-space domain Green’s function from the source s to the subsurface point (x,z). The space domain Green’s function can be decomposed at the image region to a summation of local wavenumber components. That is

$$G(x,z,s,\omega) = \sum_{\theta} G(x,z,\theta,s,\omega),$$  \hfill (1)

where $G(x,z,s,\omega)$ is the frequency-space Green’s function and $G(x,z,\theta,s,\omega)$ is its local-angle component at $\theta$. In the 3D case, $\theta$ is a two dimensional vector, $(\theta, \varphi)$, which are dip angle and azimuthal angle, respectively. Similarly, the frequency-space Green’s function from the subsurface point (x,z) to receiver r can be decomposed

$$G(x,z,r,\omega) = \sum_{\theta} G(x,z,\theta,r,\omega),$$  \hfill (2)

where $G(x,z,r,\omega)$ is the frequency-space Green’s function and $G(x,z,\theta,r,\omega)$ is its local-angle
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component at \( \theta_x \). We use the reciprocity principle in the calculation of the receivers’ Green’s function, which means we calculate the Green’s function from the receiver to the subsurface point and use it for our illumination analysis. In order to evaluate the local angle domain illumination energy distribution for a given acquisition system in a velocity model, we sum up the contributions from all the sources at the image point, for each local angle. That is

\[
D_d(x,z,\theta_{\hat{n}},\omega) = \sum_s |G(x,z,\theta_{\hat{n}},s,\omega)|^2 ,
\]

where \( D_d(x,z,\theta_{\hat{n}},\omega) \) is the directional illumination map. Then we can get the total illumination by the summation of all the angles

\[
D_{total}(x,z,\omega) = \sum_{\theta_{\hat{n}}} \sum_s |G(x,z,\theta_{\hat{n}},s,\omega)|^2 .
\]

In order to evaluate the aperture and propagation effects of the given acquisition geometry on energy distribution for a specific pair of incident/receiving angles, we use unit impulse as the source at both source and receiver points for the entire acquisition configuration and assume a unit scattering coefficient at each space point. For a given target horizon, we know the position of the horizon in the model. Then we can calculate the normal angle at every point on the horizon. In this case, we have a set of local incident angles, \( \theta_{\hat{n}} \). The corresponding receiving angles, \( \theta_x \), can be calculated by \( \theta_x = 2\theta_{\hat{n}} - \theta_{\hat{n}} \), where \( \theta_x \) is the normal angle at point \((x,z)\) on the target horizon. Then we can get a number of ADR’s on the horizon, which are defined as

\[
A(x,z,\theta_{\hat{n}},\theta_x,\omega) = \sum_s |G(x,z,\theta_{\hat{n}},s,\omega)|^2 \sum_r |G(x,z,\theta_x,r,\omega)|^2 .
\]

We can further sum up all the reflected energy to get the total ADR as follow

\[
A_d(x,z,\omega) = \sum_{(\theta_x,\theta_{\hat{n}})} E(x,z,\theta_{\hat{n}},\theta_x,\omega) .
\]

The value of the ADR map measures the dip-angle response of the acquisition system, including the source and receiver apertures, and propagation effects.

Efficient scheme of illumination analysis along the target horizon

In order to make the calculation fast, we developed a new scheme for this target oriented illumination analysis. Usually, the aperture for adjacent shots will have a big overlap area. If we calculate the Green’s function on each receiver location for each shot, there will be repeated calculation. So our scheme is to calculate all the Green’s function on the target horizon in the acquisition area and save them. Then we loop over the shots and read in the Green’s function used in the aperture and form the illumination. When we finish one shot, we keep the Green’s functions which are located in the aperture of next shot, and read in the Green’s functions which are not used in last shot. This will save a lot of I/O time. We summarized the scheme in Figure 1, which shows how we do the fast illumination scheme on the target horizon.

![Flowchart of the illumination scheme](image1)

Figure 1: Flowchart of the illumination scheme

Numerical examples

To demonstrate the application of the illumination analysis, we first calculated several numerical examples using the 3D SEG/EAGE salt velocity model. This example simulates the illumination condition of the 45-shot data set. The grid size for the model is 676, 676 and 210 cells along the x, y and z axes, respectively, with 20 m for both the horizontal and depth intervals. Figure 2 is a velocity slice from the model. The orange line indicates the picked horizon in the subsalt area. Next, we calculated the DI and ADR maps on the target horizon. We calculated angle domain DI and ADR maps with a set of 25 incident angles. In Figure 3, we only selected three angle components to
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plot. DI maps only show how much energy can be propagated to the target horizon. ADR maps can estimate how much energy can be received by the given acquisition system. As a result, ADR will be more consistent with the image amplitude. After a summation, we can get the total ADR as shown in Figure 4.

![Figure 3: Angle domain DI maps (left hand side) and the corresponding ADR maps (right hand side) for the target horizon. The incident angles for these DI and ADR maps are (0°,0°), (60°,0°) and (40°,225°) from top to the bottom.](image)

Figure 4: Total ADR map on the target horizon

![Figure 5: A field data set (upper: velocity model, lower: migration image)](image)

Figure 6: Subsalt target horizon in the field data model

Then we work on a field data example model. Figure 5 shows the velocity cube and RTM image, which has a couple of salt bodies in it. We can see the image amplitude is not balanced in the sub-salt area due to the illumination problem, in such areas illumination analysis will be helpful for future acquisition design. Figure 6 shows the depth location of a given target horizon. We generated two different acquisition systems for testing (the aperture size for these two system are the same but the shots are located on different sides of the aperture). Figures 7 and 8 are the DI and ADR maps for Acquisition 1 on the target horizon. Figure 9 is the total ADR. Figures 10 and 11 are angle domain ADR and total ADR maps on the target horizon for Acquisition 2. From these result, we can see that the illuminated areas are different for different acquisition systems. As a result, we can use the illumination results as a guide when we design certain acquisition geometries to illuminate certain target areas. After we do the migration,
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we can also use these maps to compensate for the image amplitudes along the target horizon.

Figure 7: DI maps for the target horizon. The incident angles for these three DI maps are (0°,0°), (60°,0°) and (40°,225°).

Figure 8: ADR maps on the target horizon. The incident angles for these three DI maps are (0°,0°), (60°,0°) and (40°,225°).

Figure 9: Total ADR on the target horizon

Conclusions

We developed an efficient method of angle domain illumination analysis along a target horizon. The DI and ADR maps on the target horizon can be very useful for acquisition design and for the interpreter when the image amplitude is unreliable on the horizon. Numerical examples of the 3D SEG/EAGE salt model and a field data model illustrated the validity and efficiency of the new scheme. For further work, we may develop an amplitude compensation method to generate AVO friendly angle gathers.

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EDITED REFERENCES
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