Limits on resolution enhancement for multicomponent data
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Summary
Two methods have recently been published for carrying out nonstationary spectral broadening (and narrowing) of PS data after it has been mapped into the PP time domain. We present a study which investigates these two papers by Bansal & Matheney (2010) and by Gaiser (2011) (see also Gaiser et al., 2011a,b).
The two approaches differ in purpose, in method of PS-to-PP time mapping, in proposed spectral corrections, and in methods for applying those corrections. In the context of comparing these two studies we add some clarification of fundamental resolution issues and illustrate our points with simple synthetic seismograms.

Introduction
A very common observation, and shortcoming, in converted-wave data is its poor resolution compared to PP data. Even after squeezing PS data from PS to PP time, the frequency content of the PS data is typically not as high as P-wave data (except for some important shallow reflector, near-surface exceptions). In the absence of attenuation we know that shear waves should provide better resolution than P-waves because the lower S-velocities have smaller spatial wavelengths (for the same temporal frequency) than the P-wavelengths. The fact that the highest frequencies in the PS data are not observed to be as high as in the PP data has typically been attributed to the fact that Qs has a more severe attenuative effect on the shear waves than the effect of Qp on the P-waves.

Recently Bansal & Matheney (2010) (hereafter referred to as Paper I) published a method for equalizing the wavelets in PS data after squeezing the time coordinate from PS time to PP time in order to prepare the data for inversion since a time-stationary wavelet is assumed to exist by inversion algorithms. Their method can generate some enhancement of the frequency content of the PS data after it is squeezed to PP time but this is not surprising since it involves a controlled form of time-varying spectral whitening which is an industry-standard method of trying to extract as much resolution out of data as possible by whitening its amplitude spectrum over the existing bandwidth.

Even more recently Gaiser (2011) (hereafter referred to as Paper II) has suggested that we have all been underestimating the true resolving power of much PS data. According to Gaiser, we have not been achieving the true resolution of the PS data after squeezing it to PP time (especially land PS data) because we have not been recognizing that the “wavelet compression during transformation appears to be more sensitive to average rather than to interval properties” and that “average velocity properties…cause the wavelet distortion” which his work is designed to correct (Gaiser, 2011). In contrast to Paper I’s conventional method of whitening existing temporal frequencies in the PS data, Gaiser’s method involves an unorthodox mapping of amplitude spectra from their native frequencies to higher frequencies, some of which may exceed the maximum frequency of the original data. If Gaiser’s point is correct, it would certainly be important since it would provide an instant method of getting better resolution from many PS datasets.
The potential improvement in resolution from Gaiser’s method motivated us to examine his argument in detail. The analysis presented here of some basic concepts of domain mapping, resolution, and wavelength preservation leads us to question some of Gaiser’s statements and to show that Bansal & Matheney’s approach is basically sound.

We start our analysis by describing two methods of mapping data from PS time to PP time. One method depends on interval velocity ratios and the other depends on average velocity ratios. Confusion between Paper I and Paper II methods starts here because, contrary to what Paper II states, the PS to PP time conversion can depend either on interval or average velocity ratio. We find that doing the PS to PP time conversion with locally constant average velocity ratios causes confounding wavelet distortions that do not occur with the true average velocity ratios or with interval velocity ratios.

Two methods of squeezing PS data from PS to PP time
There are at least two methods that can be used to map PS data from PS time to PP time. One of them constructs the squeezed PS trace using the interval Vp/Vs ratio, γint, and the other constructs the squeezed PS trace using a Vp/Vs ratio that is averaged from the surface, γ0. The interval Vp/Vs-based method uses the factor 2/(1+γint) to do the mapping sample by sample within each constant interval. The average Vp/Vs-based method uses the factor 2/(1+γ0) to do the mapping sample by sample across the entire trace. The factor 2/(1+γ) is the ratio of traveltimes, tP/S, either across a region of constant γint, in the case of the γint-based method, or averaged from the surface, in the case of the γ0-based method.

In the case of the γint-based method, let us assume that the γint model is blocky: i.e. Vp/Vs is constant within a block of time samples. So we start with a set of γint values that are used to map time samples within blocks of the original PS trace to blocks of samples in the squeezed PS trace in the following way. Beginning at the top, the squeezed PS trace is constructed interval by interval by first squeezing the sample interval, ΔtP/S, of the top interval of the original PS trace by a constant factor, 2/(1+γint), interpolating the squeezed samples within that interval to the desired ΔtP of the output trace, and then placing those interpolated...
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samples at the top of the squeezed PS trace. This procedure would be repeated for the samples within the second constant \( \gamma_{\text{int}} \) interval and those squeezed and interpolated samples would be pasted below the top layer of the squeezed PS trace. And then the third layer would be squeezed, interpolated and pasted below the second layer of samples, and so on to the bottom of the trace.

In the case of the \( \gamma_0 \)-based method, we start with a \( \gamma_0 \) model which will typically vary sample by sample down the trace (except for the top layer if the underlying \( \gamma_{\text{int}} \) model is blocky). Therefore, the squeeze factor \( 2/(1+\gamma_0) \) defines a point-by-point mapping of samples in the original PS trace to samples in the squeezed PS trace. Since \( \gamma_0 \) typically varies sample by sample, the amount of squeezing varies sample by sample as well. Interpolation will be required to map from constant \( \Delta t_{\text{ps}} \) samples in the original trace to constant \( \Delta t_{\text{pp}} \) samples in the squeezed PS trace.

Notice that the method of obtaining the \( \gamma_{\text{int}} \) or \( \gamma_0 \) models has not been described so far. It is important to distinguish the method of obtaining the \( V_p/V_s \) model from the method of performing PS-to-PP time mapping. Gaiser (1996) describes a cross-correlation based method that naturally yields a \( \gamma_0 \) model, but one could also employ a Dix inversion method to convert the \( \gamma_0 \) values to a \( \gamma_{\text{int}} \) model. While Paper II does calculate \( \gamma_{\text{int}} \) for at least one point, it appears to use only the average \( V_p/V_s \) based method to compress the PS traces. Paper I describes a method of registering PP-horizons to PS-horizons that naturally leads to a blocky \( \gamma_{\text{int}} \) model, and it appears that they have used this interval \( V_p/V_s \) model to do their PS-to-PP mapping.

But it would be a simple procedure to integrate Paper II’s \( \gamma_{\text{int}} \) model to obtain a \( \gamma_0 \) model and then do the mapping by the \( \gamma_0 \) mapping method. The important point is that the method of mapping from PS to PP time does not need to be tied to the method of obtaining the \( V_p/V_s \) model.

The two mapping methods are compared in Figure 1, which shows a simple model and the associated \( \gamma_{\text{int}} \) and \( \gamma_0 \). Also shown are a PP trace and a PS trace compressed to PP time by each of the methods described above. It is clear that both methods result in identical wavelets that are compressed in time relative to the PP wavelets, and that the arrival time of each event in PP time is correct. Both methods also result in nonstationary wavelets. However we note that the wavelets are stationary within a constant-\( \gamma_{\text{int}} \) interval, so that their frequency bandwidth after compression is governed by \( \gamma_{\text{int}} \) rather than \( \gamma_0 \).

One point of interest is the asymmetric compressed wavelet of the second event (i.e., second pane in Fig. 1c). This is an artifact of the discontinuity in \( \gamma_{\text{int}} \) and in the derivative of \( \gamma_0 \), and should not be present. One possible approach is to modify \( \gamma \) so that it is constant in a region about each event. This yields symmetric wavelets throughout the section as shown in Figure 2.

However this procedure has an unintended consequence as well, namely that wavelets below the first layer possess a different frequency bandwidth when compressed by a locally constant \( \gamma_0 \), which we will denote \( \gamma_{\text{ps}} \). In fact they are somewhat narrower than before which makes it seem that using \( \gamma_{\text{ps}} \) would be a useful way to enhance resolution. This is not the case, as we show in the next section, but first we will describe the wavelets in Figure 2 more precisely.

If the original wavelet (in either PP or PS traces) is characterized by a dominant frequency of \( f_0 \), then it can be shown (Ursenbach et al., 2012) that the dominant frequency of the PS wavelet squeezed to PP time using \( \gamma_0 \) (or \( \gamma_{\text{int}} \)) is \( 2f_0/(1+\gamma_{\text{int}}) \), consistent with Eq.(3) of Paper II, while that of the wavelet squeezed using \( \gamma_{\text{ps}} \) is \( 2f_0/(1+\gamma_0) \) or, for clarity, \( 2f_0/(1+\gamma_0) \). Thus it is only when \( \gamma_0 \) is employed in mapping that wavelets would experience the distortion by average velocity properties described in Paper II.

What is the importance of understanding how the PS-to-PP domain transformation is carried out, and whether one of these methods is better than another? We address this question in our next section.

Wavelength = Resolution

In the last section we showed that the way in which wavelet frequency content varies with time in squeezed PS traces depends upon the manner in which the domain transformation is carried out. This is important because frequency is related to wavelength, and wavelength is a fundamental measure of the resolving power of a wavefield. The P- and S-wavefield wavelengths in layer \( i \), \( \lambda_{pi} \) and \( \lambda_{si} \), have a clear physical meaning. It can also be shown (Ursenbach et al., 2012) that a natural definition of \( \lambda_{ps} \) for discussion of resolution is the harmonic average,

\[
\frac{1}{\lambda_{ps}} = \frac{1}{\lambda_{pi}} + \frac{1}{\lambda_{si}} / 2.
\]

The significance of this result is that, as discussed in Gaiser (1996), resolution is fundamentally related to wavelength. For instance if \( V_p/V_s = 2 \) for all layers, then before mapping from SS time to PP time, the SS signal will possess twice the resolving power of a PP signal with the same frequency bandwidth, because \( \lambda_{si} \) is half of \( \lambda_{pi} \). Thus if a PP signal can detect a 20m thick layer, then an SS signal can detect a 10m layer, and a PS signal a ~13.3m layer.

After mapping to PP time the SS signal would have an effective velocity of \( V_p \) instead of \( V_s \), by virtue of it having been shifted in time, and the frequency of its time trace would now be \( 2f_0 \). The doubling of the frequency does not mean however that its resolution has been doubled from what it was before the mapping, for the fact that its wavelength, \( (2V_s)/(2f_0) = \lambda_{si} \), is unchanged means that its resolution has been preserved across the domain mapping.

What the double frequency does tell us is that the resolution is still double that of the PP signal, for this can now be discerned from frequencies as well as wavelengths because the signals are now in the same time domain. The essential idea to take away from this is that resolution is fundamentally determined by wavelength, and this
resolution can be preserved through various domain transformations, but it cannot be fundamentally increased once acquisition is complete.

One of the valuable points made in Paper II is that the frequency of domain-transformed PS wavelets should be corrected in such a way that wavelengths are correct. We would say that a wavelength is correct if it maintains the true resolving power of the original PS signal. After transforming to PP time the effective velocities of the PS signals become equal to P-wave velocities, and if the transformation employs \( \gamma_{0i} \), then the frequency becomes \( f \frac{1}{V_0} (1 + \gamma_{0i}) / 2 \). Thus the implied wavelength for a PS event from the bottom of interval \( i \) is \( \lambda_{\text{squeezed}} = 2V_0 f / [f \frac{1}{V_0} (1 + \gamma_{0i})] \). As pointed out in Paper II, this is not the correct wavelength, and Paper II thus proposes a correction. We point out here though that if the transformation employs either \( \gamma_0 \) or \( \gamma_{\text{int}} \) then the frequency becomes \( f \frac{1}{V_0} (1 + \gamma_{\text{int}}) / 2 \). Thus the implied wavelength is \( \lambda_{\text{squeezed}} = 2V_0 f / [f \frac{1}{V_0} (1 + \gamma_{\text{int}})] = 2 / [f \frac{1}{V_0} (1 + V_{\text{int}}/V_0)] \), which is precisely the wavelength that correctly describes the PS signal’s resolution (i.e., \( \lambda_{\text{PS}} \) in equation 1).

In Figure 2 the result of squeezing with \( \gamma_0 \) produced narrower wavelets than for \( \gamma_0 \) or \( \gamma_{\text{int}} \). This would perhaps make the use of \( \gamma_0 \) tempting. To illustrate the danger of this though, we show in Figure 3 the effect of each transformation on tuned wavelets. Figure 3a is similar to the third event in Figure 2c, but the single event has been replaced by two closely spaced wavelets of opposite sign. This models the important case of an embedded thin layer. In Figure 3b we show the \( \gamma_{\text{int}} \)-squeezed PS signal along with the “ideal” result, in which the true reflectivity has been squeezed to PP time and then convolved with a wavelet whose dominant frequency is \( (1 + \gamma_{\text{int}}) / 2 \) times that of the original PS wavelet. We see that the two signals are identical, showing that the resolving power of the original PS signal has been preserved. In Figure 3c we show the \( \gamma_0 \)-squeezed PS signal along with the “ideal” result in which the true reflectivity has been squeezed to PP time and then convolved with a wavelet whose dominant frequency is \( (1 + \gamma_0) / 2 \) times that of the original PS wavelet. Now we see that the two signals differ significantly. The ideal synthetic shows that this higher frequency should begin to resolve the tuned events. However the squeezing process produces a signal which implies that the tuned events are closer to each other than they actually are; in other words, the underlying reflectivity has been distorted by the squeezing process.

Thus a distinct advantage of performing PS-to-PP domain transforms with \( \gamma_{\text{int}} \), as in Paper I, or with \( \gamma_0 \), is that the wavelengths automatically assume their correct values, as demonstrated in Figure 3. Then no correction is necessary.

**Conclusions**

We have demonstrated that a PS to PP time mapping that is performed correctly preserves both wavelength and resolution of the underlying PS signal. So contrary to Paper II, we would say that we have not been underestimating the true resolving power of PS data. The correct PS resolution is obtained when PS-to-PP time mapping is carried out using interval \( V_P/V_S \) ratios or exact average \( V_P/V_S \) ratios as we have shown for a blocky model. If one uses locally constant average \( V_P/V_S \) ratios then the frequency is scaled incorrectly and wavelengths are not preserved. Although the incorrect scaling may in some cases appear to increase resolution, we have shown that for tuned wavelets it can result in distortion of the underlying reflectivity.

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**Figure 1:** (a) Interval and average \( V_P/V_S \) values in depth for a simple model. The locations of four reflectors are indicated by symbols. (b) A PP trace (black line), a PS trace compressed to PP time by a method that uses interval \( V_P/V_S, \gamma_{\text{int}} \) (green line), and a PS trace compressed to PP time by a method that uses average \( V_P/V_S, \gamma_0 \) (dashed red line). (c) Close-ups of each of the four events in (b).
Figure 2: (a) Interval and average $V_P/V_S$ values in PS time for the same model as in Figure 1, but with $V_P/V_S$ values held approximately constant in a wavelet-sized region around each event. The PS times of four reflections are indicated by symbols. (b) A PP trace (black line), a PS trace compressed to PP time by a method that uses locally constant interval $V_P/V_S$, $\gamma_{int}$ (green line), and a PS trace compressed to PP time by a method that uses locally constant average $V_P/V_S$, $\gamma_0$ (red line). (c) Close-ups of each of the four events in (b).

Figure 3: a) This panel is similar to the third event in Figure 2c, but the single event has been replaced by a pair of tuned events of opposite sign, located at 0.3367 s and 0.3567 s. b) The green line from Part a displayed together with a convolution of the reflectivity with a Ricker wavelet in which the dominant frequency has been multiplied by $2/(1+\gamma_{int})$. Their exact coincidence shows that reflectivity has been preserved in squeezing with $\gamma_{int}$. c) The red line from Part a displayed together with a convolution of the reflectivity with a Ricker wavelet in which the dominant frequency has been multiplied by $2/(1+\gamma_0)$. The difference between these lines shows that reflectivity has not been preserved in squeezing with $\gamma_0$. 
EDITED REFERENCES

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REFERENCES


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