Kirchhoff or wave equation?

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Summary

Wave equation (or wave field extrapolation) techniques have been used in industry for the past few years, with the purpose to improve the accuracy of 3D depth imaging over the conventional Kirchhoff migration. However, on many field data examples using different wave equation implementations from different processing shops, we have seen high-quality imaging from both Kirchhoff and wave equation techniques. In the near future, we see that both Kirchhoff and wave equation implementations will continue to serve high fidelity imaging needs and also complement each other in terms of strengths and weaknesses. The benefits of wave equation methods would be significant and overwhelming, perhaps, when we have advanced the acquisition technology to obtain true 3D marine data in the future.

Introduction

Kirchhoff depth imaging has been used in the past decade to produce volume pre-stack images in the complex area. Despite many practical advantages, such as run-time efficiency, amplitude treatment, steep dip accuracy and output image gathers, most Kirchhoff algorithms have difficulty to handle multiple arrivals between a surface position (either source or receiver) and a subsurface point. Wave equation methods, on the other hand, image multiple arrivals properly through downward continuation of wave fields. However, other approximations, such as steep-dip limit on finite difference methods or velocity simplification on mixed domain (Fourier-space) methods, have become so dominant that the artifacts caused by these approximations could overshadow the benefits of wave equation migration. Then there is always the issue of subsurface illumination- do we have recorded all the multiple arrival signals in our current acquisition? What offsets and surface locations would the reflections below the complex overburden be recorded? How do we update the velocity model using the wave equation gathers, common shot or common angle?

These practical issues often made us use both Kirchhoff and wave equation methods in the same 3D area, at multiple times during the course of prospect evaluation or field development, and sometimes, on multiple 3D data sets. Until the marine acquisition technology to acquire true 3D data becomes available, we will probably have to continue using both techniques in practice.

Kirchhoff imaging

Major difference between many Kirchhoff migrations is the travel time calculation. Following table lists four different travel time schemes commonly used in Kirchhoff imaging.

methods	**issues**
Eikonal	first arrival; should not be used in salt related imaging
wave front	continuous travel time coverage; artifacts around and direct beneath salt
dynamic ray	images on certain offsets and locations; single arrival only
beams	handle multiple arrivals to some extent; sufficient p sampling difficult

Table 1: Travel time computations used in Kirchhoff imaging.

High quality travel time computation, such as dynamic ray tracing, is essential to the accuracy of Kirchhoff migration to obtain clean, steep-dip, sub-salt images. Figures 1 and 2 show comparison between a Kirchhoff algorithm and a wave equation method.

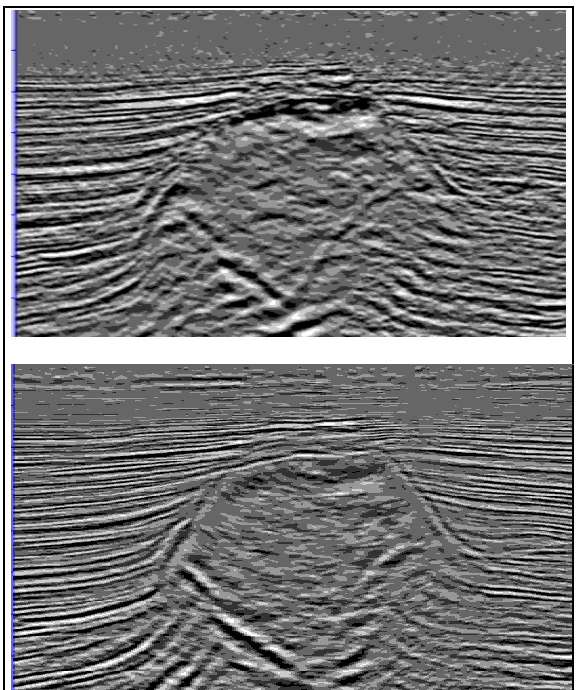

Figure 1: Cross-line images of 3D pre-stack wave-equation (top) and Kirhoff (bottom), Gulf of Mexico.

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In this case, a high quality Kirchhoff will be just as good as, or perhaps better than, the wave equation result. Sediment truncations are better imaged in the Kirchhoff migration (Figure 1). Sharper and more continuous sub-salt events can be seen in the Kirchhoff image (Figure 2).

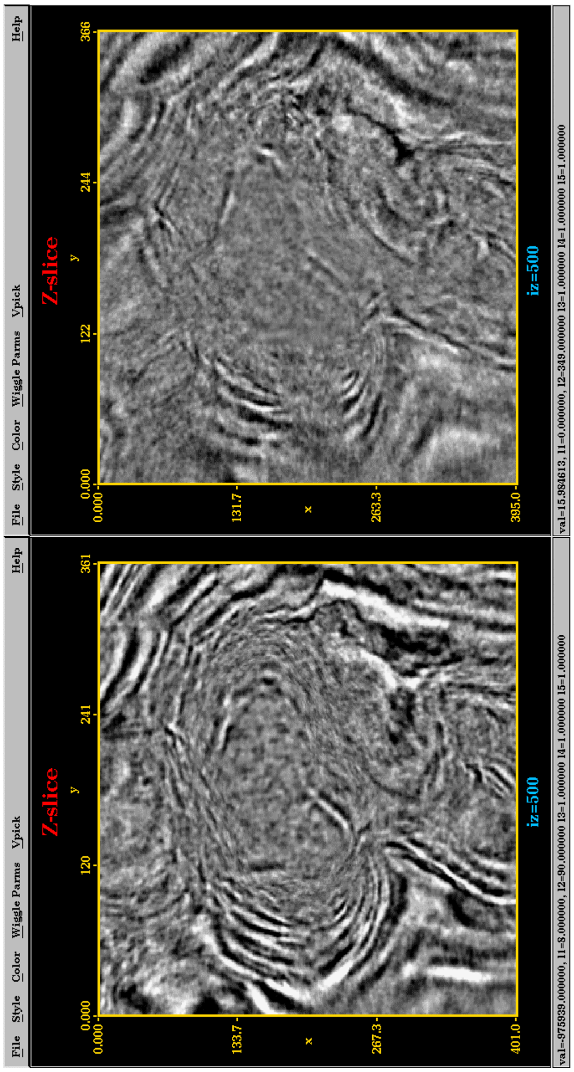


Figure 2: Sub-salt depth slices of 3D pre-stack wave-equation (top) and Kirchhoff (bottom), Gulf of Mexico.

Figure 3 shows the same Kirchhoff algorithm applied to SMAART JV's Sigsbee2a data set. Though definitely not as good as some of the wave equation results we have seen in the past SEG presentations, it is perhaps among one of the best Kirchhoff results that show reasonable sub-salt imaging quality for this 2D synthetic data set.

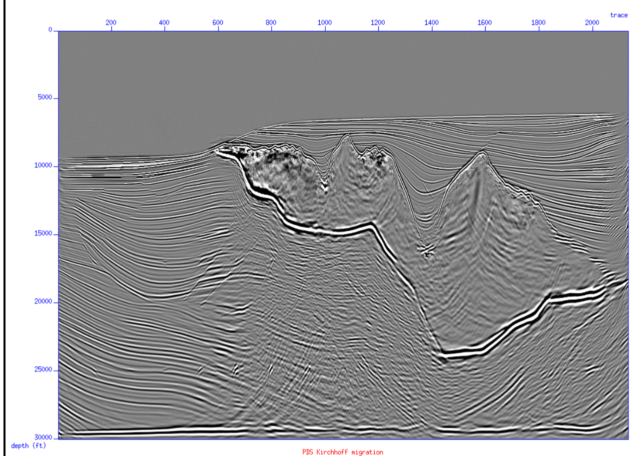
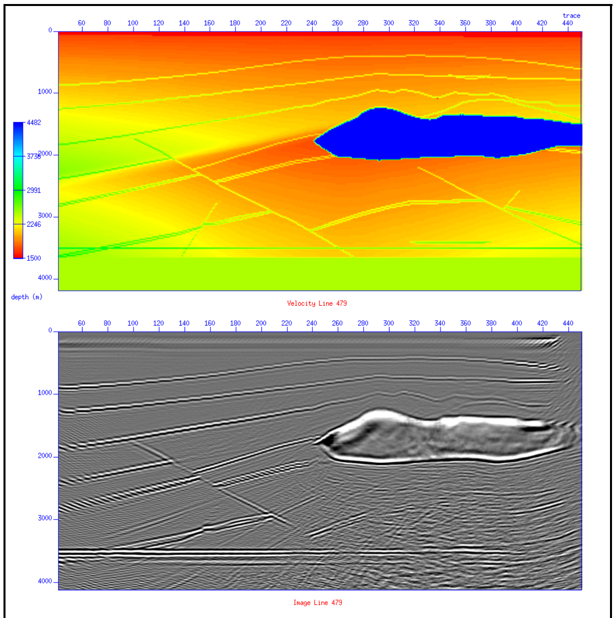


Figure 3: 2D pre-stack depth Kirchhoff migration of Sigsbee2a data set (data courtesy of SMAART JV).

The Kirchhoff migration was also applied to the SEG/EAGE salt 3D C3-NA (narrow azimuth) data set, as shown in Figure 4. Once again, reasonable base salt and sub-salt images are obtained in this Kirchhoff run, though we have seen better wave equation results before.

Wave equation imaging

There are two major categories of wave equation migrations, the common-azimuth (or narrow-azimuth) method and the common-shot method. Common-azimuth migration uses Fourier-Space mixed domains to downward continue wave field. Common-azimuth migration has steep dip accuracy since the extrapolation in done in the Fourier domain. Limitations or assumptions of common azimuth methods are 1) wave field stays in the same azimuth when downward extrapolated through lateral varying velocity media and 2) several constant reference velocities must be used in each step of the Fourier-domain extrapolation and errors are difficult to compensate completely later using the space domain finite-difference methods. Common-shot migration, on the other hand, is an inverse process of how the data is acquired in the field, fits the physics and handles velocity variation nicely by wave field extrapolation in the space domain. However, common problems of finite difference dispersion, steep dip approximation (when one-way wave equation is used) and huge amount of CPU run time made the use of common-shot migration less appealing than it should be.



Fiure4: Line 479 of 3D SEG/EAGE salt model. Velocity secion (top) and Kirchhoff image (bottom). No multiple attenuation applied before or after migration.

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Velocity analysis and use of the image gathers may be another factor when deciding Kirchhoff or wave equation. Almost all the Kirchhoff migrations will output image gathers which can be used for velocity analysis and AVO work. Wave equation migration often outputs only the stacked images. Sensitivity of image quality on velocity model may also affect the decision on when to use Kirchhoff and when to use wave equation.

After examining pros and cons of both Kirchhoff and wave equation methods, and seeing many examples of fair comparisons (good wave equation versus good Kirchhoff), it appears that we saw significant improvement of wave equation over Kirchhoff on the synthetics, but less or no improvement (sometimes worse) in the field data sets. Besides the above mentioned issues with the wave equation and the Kirchhoff methods, the signal to noise ratio in the input data is, very often, the most important and deciding factor on the quality of depth migration. The signal to noise ratio is affected by the signal strength (depending on sufficient illumination of the area beneath the complex overburden) and the noise level (multiples, acquisition noise, etc.). Generally, Kirchhoff methods (with input and output mutes, spatial aperture control, travel time illumination) are more of local operators, whereas wave equation techniques apply wave extrapolation globally. Therefore, the wave equation methods are less immune to the low signal to noise ratio input, thus less appealing in the practice than in the synthetic models.

To improve the illumination, we have seen multiple vintages of 3D data being shot in different orientations. In most cases of true 3D structure, we often use different 3Ds to image different parts of the reservoirs.

Following table summarizes where Kirchhoff and wave equation methods would be preferred.

geology\signal-noise ratio	bad	poor	good
complex	A	W/K	W
moderate	A	K	W
mild	A	K	K/W

Table 2: Choice of Kirchhoff and wave equation. When signal-noise ratio is bad (no illumination), better acquisition (A) should be considered. Poor S/N area with moderate to large structures, Kirchhoff (K) is preferred. Good signal-noise ratio with moderate to small features, wave equation (W) should be used.

New advancement of true 3D marine data acquisition is needed to better illuminate subsurface and provides better input to the advanced 3D multiple attenuation methods. The aerial layout of hydrophones around the air gun sources would certainly benefit, both accuracy and efficiency wise, the common shot wave equation migration and the other shot-based processing procedures, which shall make the choice of migration methods easier.

Conclusions

Choice of migration methods depends on both geological features (dips and velocity variation) and signal to noise ratio. With the current steamer type of 3D marine acquisition, Kirchhoff techniques remain one of main production tools to output large volume of 3D images over the entire 3D area, whereas wave equation methods can be used in some local areas where the overburden (velocity and structure) is extremely complex and if input data has sufficient signal to noise ratio. Acquisition technology has to catch up with the development of wave equation imaging in the near future.

Acknowledgments

We would like to thank BP and its partners for permission to use the field data in this paper. Thanks to David Warraven of Anadarko, SMAART JV and Paul Docherty of Fairfield for providing synthetic data sets.